**LECTURE 12**

# On Deriving the Effective Interaction from the QCD-Lagrangian

H. C. Pauli

*Max-Planck-Institut für Kernphysik*
*D-69029 Heidelberg*

The canonical front form Hamiltonian for non-Abelian SU(N) gauge theory in 3+1 dimensions is mapped non-perturbatively on an effective Hamiltonian which acts only in the Fock space of a quark and an antiquark. The approach is based on the novel method of iterated resolvents and on discretized light-cone quantization, driven to the continuum limit. It is free of the usual Tamm-Dancoff truncations of the Fock space, rather the perturbative series are consistently resumed to all orders in the coupling constant. Emphasis is put on dealing with the many-body aspects of gauge field theory. Important is that the higher Fock-space amplitudes can be retrieved self-consistently from these solutions.

## 1.    THE STRUCTURE OF THE HAMILTONIAN

In canonical field theory the four components of the energy-momentum vector $P^\nu$ commute and are constants of the motion. In the front form of Hamiltonian dynamics [1] they are denoted [2] by $P^\nu = (P^+, \vec{P}_\perp, P^-)$. Its spatial components $\vec{P}_\perp$ and $P^+$ are independent of the interaction and diagonal in momentum representation. Their eigenvalues are the sums of the single particle momenta, $P^+ = \sum p^+$ and $\vec{P}_\perp = \sum \vec{p}_\perp$. Each single particle has four-momentum $p^\mu = (p^+, \vec{p}_\perp, p^-)$ and sits on its mass-shell $p^\mu p_\mu = m^2$. Each particle state "$q$" is then characterized by six quantum numbers $q = (p^+, \vec{p}_\perp, \lambda; c, f)$: the three spatial momenta, helicity, color and flavor. The temporal component

| n | Sector | 1 qq̄ | 2 gg | 3 qq̄ g | 4 qq̄ qq̄ | 5 gg g | 6 qq̄ gg | 7 qq̄ qq̄ g | 8 qq̄ qq̄ qq̄ | 9 gg gg | 10 qq̄ gg g | 11 qq̄ qq̄ gg | 12 qq̄ qq̄ qq̄ g | 13 qq̄ qq̄ qq̄ qq̄ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | qq̄ | | · | ⟨v⟩ | · | · | · | · | · | · | · | · | · | · |
| 2 | gg | · | | ⟨v⟩ | · | ⟨v⟩ | · | · | · | · | · | · | · | · |
| 3 | qq̄ g | ⟨v⟩ | ⟨v⟩ | | ⟨v⟩ | · | ⟨v⟩ | · | · | · | · | · | · | · |
| 4 | qq̄ qq̄ | · | · | ⟨v⟩ | | · | · | ⟨v⟩ | · | · | · | · | · | · |
| 5 | gg g | · | ⟨v⟩ | · | · | | ⟨v⟩ | · | · | ⟨v⟩ | · | · | · | · |
| 6 | qq̄ gg | · | · | ⟨v⟩ | · | ⟨v⟩ | | ⟨v⟩ | · | · | ⟨v⟩ | · | · | · |
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| 8 | qq̄ qq̄ qq̄ | · | · | · | · | · | · | ⟨v⟩ | | · | · | · | ⟨v⟩ | · |
| 9 | gg gg | · | · | · | · | ⟨v⟩ | · | · | · | | ⟨v⟩ | · | · | · |
| 10 | qq̄ gg g | · | · | · | · | · | ⟨v⟩ | · | · | ⟨v⟩ | | ⟨v⟩ | · | · |
| 11 | qq̄ qq̄ gg | · | · | · | · | · | · | ⟨v⟩ | · | · | ⟨v⟩ | | ⟨v⟩ | · |
| 12 | qq̄ qq̄ qq̄ g | · | · | · | · | · | · | · | ⟨v⟩ | · | · | ⟨v⟩ | | ⟨v⟩ |
| 13 | qq̄ qq̄ qq̄ qq̄ | · | · | · | · | · | · | · | · | · | · | · | ⟨v⟩ | |

Fig. 1. — The Hamiltonian matrix for a SU(N)-meson and a harmonic resolution $K = 4$. Only vertex diagrams are included. Zero matrices are marked by a dot (·).

$P^- = 2P_+$ depends on the interaction and is a complicated non-diagonal operator [4, 6]. It propagates the system in the light-cone time $x^+ = x^0 + x^3$. The contraction of $P^\mu$ is the operator of invariant mass-squared,

$$P^\mu P_\mu = P^+ P^- - \vec{P}_\perp^2 \qquad \equiv H_{\mathrm{LC}} \equiv H \ . \tag{1}$$

It is Lorentz invariant and refered to somewhat improperly but conveniently as the 'light-cone Hamiltonian' $H_{\mathrm{LC}}$, or shortly $H$. One seeks a representation in which $H$ is diagonal $H|\Psi\rangle = E|\Psi\rangle$. Introductory texts [4, 5, 6] are available.

The (light-cone) Hamiltonian is split for convenience into four terms: $H = T + V + F + S$. The kinetic energy $T$ is diagonal in Fock-space representation, its eigenvalue is the free invariant mass squared of the particular Fock state. The vertex interaction $V$ is the relativistic interaction *per se*. It has no diagonal matrix elements, is linear in $g$ and changes the particle number by 1. The instantaneous interactions $F$ and $S$ are consequences of working in the light-cone gauge $A^+ = 0$. They are proportional to $g^2$. By definition, the seagull interaction $S$ conserves the particle number and the fork interaction $F$ changes it by 2. As a consequence of working in light-cone representation and in the light cone gauge, the vacuum state has no matrix elemens with any of the Fock states: The vacuum *does not fluctuate*.

The eigenvalue equation $H|\Psi\rangle = E|\Psi\rangle$ stands for an infinite set of coupled integral equations [6]. The eigenfunctions $|\Psi\rangle$ are superpositions of all Fock-

space projections like $\langle q\bar{q}|\Psi\rangle$ or $\langle q\bar{q}\,g|\Psi\rangle$. However, if one works with discretized light-cone quantization (DLCQ [3]), one deals with a *finite set* of coupled matrix equations. As consequence of periodic boundary conditions the single particle momenta are discrete and the Fock states are denumerable and orthonormal. The Hamiltonian matrix is illustrated in Fig. 1 for the *harmonic resolution* $K \equiv P^+L/(2\pi) = 4$. Its rows and columns are the Fock states which are grouped into the Fock-space sectors $q\bar{q}$, $q\bar{q}\,g$, ... , denumerated for simplicity by $n = 1, 2, \ldots, 13$. Within each sector one has many Fock states, which differ by the helicities, colors, and momenta of the single particles, subject to fixed total momenta $P^+$ and $\vec{P}_\perp$. Since $P^+$ has only positive eigenvalues and since each particle has a lowest possible value of $p^+$, the number of particles in a Fock state is limited for any fixed value of $K$: The number of Fock space sectors is *finite.* The transversal momenta $\vec{p}_\perp$ can take either sign. Their number must be regulated by some convenient cutoff, (Fock-space regularization, *f.e.* see [2]). The most remarkable property of the Hamiltonian matrix is its block structure and its sparseness. Most of the blocks are zero matrices, marked by a dot ($\cdot$) in the figure. Only those blocks in Fig. 1 denoted by the graphical symbol of a vertex are potentially non-zero, with the actual matrix elements tabulated elsewhere [4, 6].

Here then is the problem, the bottle neck of any Hamiltonian approach in field theory: The dimension of the Hamiltonian matrix increases exponentially fast. Suppose, the regularization procedure allows for 10 discrete momentum states in each direction. A single particle has then about $10^3$ degrees of freedom. A Fock-space sector with $n$ particles has then roughly $10^{n-1}$ different Fock states. Sector 13 with its 8 particles has thus about $10^{21}$ and sector 1 ($q\bar{q}$) about $10^3$ Fock states. What one wishes instead of is to derive an effective interaction which acts only in the comparatively small $q\bar{q}$-space, like a Coulomb interaction acts only between an electron and a positron. Loosely speaking, the aim of deriving an effective interaction can be understood as reducing the dimension in a matrix diagonalization problem from $10^{21}$ to $10^3$!

The first attempt to formulate an effective interaction in field theory is the approach of Tamm and Dancoff (TDA [7, 8]). The present 'method of iterated resolvents' [9, 10] is closely related to TDA. Alternatively, one can apply a sequence of analytical and approximate unitary transformations in order to render the Hamiltonian matrix more and more 'band diagonal'. Two methods have been proposed recently, the 'similarity transform' of Glazek and Wilson, see [11], and the 'Hamiltonian flow equations' of Wegner [12]. All of these methods are under active research. Applications particularly by Trittmann *et al* [13, 14, 15], Brisudova *et al* [16, 17], Jones *et al* [18, 19], and Gubankova *et al* [20] are presented elsewhere in these proceedings. Common to most of them is that one truncates the Fock space before approximation methods are applied. No truncation of the Fock space is needed with the method of iterated resolvents, as to be shown below.

## 2. THE METHOD OF ITERATED RESOLVENTS

The method of effective interactions is a well known tool in many-body physics which in field theory is known as the Tamm-Dancoff-approach [7, 8]. The method is ideally suited for a field theory, because Fock-space sectors $|i\rangle$ appear in the most natural way. The Hamiltonian matrix can be understood as a matrix of block matrices, whose rows and columns are enumerated by $i = 1, 2, \ldots N$ like in Fig. 1. The eigenvalue equation can be written as a coupled set of block matrix equations:

$$\sum_{j=1}^{N} \langle i|H|j\rangle \, \langle i|\Psi\rangle = E \, \langle n|\Psi\rangle \qquad \text{for all } i = 1, 2, \ldots, N \; . \tag{2}$$

In TDA, the rows and columns are split into a $P$-space, $P = |1\rangle\langle 1|$, and the rest, the $Q$-space. Explicitly written out, the eigenvalue equation (2) becomes a set of two coupled block matrix equations:

$$\langle P|H|P\rangle \, \langle P|\Psi\rangle + \langle P|H|Q\rangle \, \langle Q|\Psi\rangle \;=\; E \, \langle P|\Psi\rangle \; , \tag{3}$$
$$\text{and} \quad \langle Q|H|P\rangle \, \langle P|\Psi\rangle + \langle Q|H|Q\rangle \, \langle Q|\Psi\rangle \;=\; E \, \langle Q|\Psi\rangle \; . \tag{4}$$

Rewrite the second equation as $\langle Q|E - H|Q\rangle \, \langle Q|\Psi\rangle = \langle Q|H|P\rangle \, \langle P|\Psi\rangle$, and observe that the quadratic matrix $\langle Q|E - H|Q\rangle$ could be inverted to express the Q-space wave-function $\langle Q|\Psi\rangle$ in terms of $\langle P|\Psi\rangle$. But the eigenvalue $E$ is unknown at this point. One introduces therefore a redundant parameter $\omega$, and defines $G_Q(\omega) = [\langle Q|\omega - H|Q\rangle]^{-1}$. The so obtained effective interaction

$$\langle P|H_{\text{eff}}(\omega)|P\rangle = \langle P|H|P\rangle + \langle P|H|Q\rangle \, G_Q(\omega) \, \langle Q|H|P\rangle \; . \tag{5}$$

acts only in the much smaller $P$-space: $H_{\text{eff}}(\omega)|\Phi_k(\omega)\rangle = E_k(\omega) \, |\Phi_k(\omega)\rangle$. Varying $\omega$ one generates a set of energy functions $E_k(\omega)$. All solutions of the fix-point equation $E_k(\omega) = \omega$ are eigenvalues of the full Hamiltonian $H$. In fact one can find all eigenvalues of $H$ despite the fact that the dimension of $H_{\text{eff}}$ is usually much smaller than that of the full $H$ [9, 10]. The procedure is formal, since the inversion of the Q-space matrix is as complicated as its diagonalization. The adventage is that resolvents can be approximated systematically: The two resolvents

$$G_Q(\omega) = \frac{1}{\langle Q|\omega - T - U|Q\rangle} \quad \text{and} \quad G_Q^{(0)}(\omega) = \frac{1}{\langle Q|\omega - T|Q\rangle} \; , \tag{6}$$

defined once with and once without the non-diagonal interaction $U$, are identically related by $G_Q(\omega) = G_Q^{(0)}(\omega) + G_Q^{(0)}(\omega) \, U \, G_Q(\omega)$, or by the infinite series of perturbation theory. Albeit exact in principle, the Tamm-Dancoff-approach (TDA) suffers from a practical aspect: The approach is useful only if one truncates the perturbative series to its very first term. This destroys Lorentz and gauge invariance, and requires a sufficiently small coupling constant.

Truncation can be avoided by 'going backwards'. Reinterpreting the $Q$-space as the last sector $N$ (sector 13 in Fig. 1) and the $P$-space with the rest, the above equations can be interpreted as to reduce the block matrix dimension from $N$ to $N-1$, with an effective interaction acting now in the smaller space of $N-1$ blocks. This procedure can be iterated, from $N-1$ to $N-2$, and so on, until one arrives at block matrix dimension 1 where the procedure stops: The effective interaction in the Fock-space sector with only one quark and one antiquark is defined unambiguously. One has to deal then with 'resolvents of resolvents', or with iterated resolvents [10].

To be more explicit suppose one has arrived in the course of this reduction at block matrix dimension $n$, with $1 \le n \le N$. Denote the corresponding effective interaction $H_n(\omega)$. The eigenvalue problem reads then

$$\sum_{j=1}^{n} \langle i|H_n(\omega)|j\rangle\langle j|\Psi(\omega)\rangle = E(\omega)\,\langle i|\Psi(\omega)\rangle\,, \quad \text{for } i = 1, 2, \ldots, n\,. \tag{7}$$

Observe that $i$ and $j$ refer here to sector numbers, and that n refers to both, the last sector number, and the number of sectors. Now, like in the above, define the resolvent as the inverse of the Hamiltonian in the last sector

$$G_n(\omega) = \frac{1}{\langle n|\omega - H_n(\omega)|n\rangle} \tag{8}$$

$$\text{thus} \quad \langle n|\Psi(\omega)\rangle = G_n(\omega)\sum_{j=1}^{n-1}\langle n|H_n(\omega)|j\rangle\,\langle j|\Psi(\omega)\rangle\,. \tag{9}$$

The effective interaction in the $(n-1)$-space becomes then

$$H_{n-1}(\omega) = H_n(\omega) + H_n(\omega)G_n(\omega)H_n(\omega) \tag{10}$$

for every block matrix element $\langle i|H_{n-1}(\omega)|j\rangle$. Everything proceeds like in above, including the fixed point equation $E(\omega) = \omega$. But one has achieved much more: Eq.(10) is a *recursion relation* which holds for all $1 < n < N$! Since one has started from the bare Hamiltonian in the last sector, one has to convene that $H_N = H$. The rest is algebra and interpretation.

Applying the method to the block matrix structure of QCD, as displayed in Fig. 1, is particularly easy and transparent. By definition, the last sector contains only the diagonal kinetic energy, thus $H_{13} = T_{13}$. Its resolvent is calculated trivially. Then $H_{12}$ can be constructed unambiguously, followed by $H_{11}$, and so on, until one arrives at sector 1. Grouping the so obtained results in a different order, one finds for the sectors with one $q\bar{q}$-pair:

$$H_{q\bar{q}} = \quad H_1 = T_1 + VG_3V + VG_3VG_2VG_3V\,, \tag{11}$$
$$H_{q\bar{q}\,g} = \quad H_3 = T_3 + VG_6V + VG_6VG_5VG_6V + VG_4V\,, \tag{12}$$
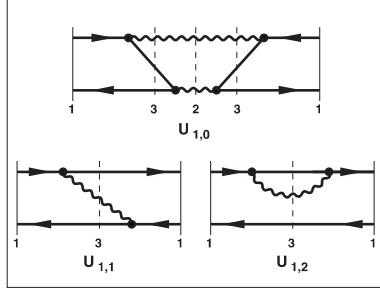$$H_{q\bar{q}\,gg} = \quad H_6 = T_6 + VG_{10}V + VG_{10}VG_9VG_{10}V + VG_7V\,. \tag{13}$$

Fig. 2. — The three graphs of the effective interaction in the $q\bar{q}$-space. — The lower two graphs correspond to the chain $U = VG_3V$, the upper corresponds to $U_a = VG_3VG_2VG_3V$. Propagator boxes are represented by vertical dashed lines, with the subscript 'n' referring to the sector numbers.

The quark-gluon content of the sectors is added here for easier identification, in line with the notation in Fig. 1. Correspondingly, one obtains for the sectors with two $q\bar{q}$-pairs

$$H_{q\bar{q}q\bar{q}} = \quad H_4 \quad = \quad T_4 + VG_7V + VG_7VG_6VG_7V \ , \tag{14}$$

$$H_{q\bar{q}q\bar{q}\,g} = \quad H_7 \quad = \quad T_7 + VG_{11}V + VG_{11}VG_{10}VG_{11}V + VG_8V \ . \tag{15}$$

In the pure gluon sectors, the structure is even simpler:

$$H_{gg} = \quad H_2 \quad = \quad T_2 + VG_3V + VG_5V \ , \tag{16}$$

$$H_{gg\,g} = \quad H_5 \quad = \quad T_5 + VG_6V + VG_9V \ . \tag{17}$$

Note that these relations are all exact.

One is left with the eigenvalue problem in the $q\bar{q}$-space,

$$\sum_{q',\bar{q}'} \langle q;\bar{q}|H_{q\bar{q}}(\omega)|q';\bar{q}'\rangle\psi_b(\omega)\rangle = M_b^2(\omega)\langle q;\bar{q}|\psi_b(\omega)\rangle \ . \tag{18}$$

By construction, the eigenvalues are identical with those of the full Hamiltonian and are enumerated by $b = 1, 2, \dots$. They are the invariant mass$^2$ of a physical particle and its intrinsic excitations. The corresponding eigenfunctions $\langle q;\bar{q}|\psi_b(\omega)\rangle$ represent the normalized projections of $|\Psi\rangle$ onto the Fock states $|q;\bar{q}\rangle = b_q^\dagger d_{\bar{q}}^\dagger|vac\rangle$. The effective Hamiltonian as given by Eq.(11) has two types of interactions which are illustrated diagrammatically in Fig. 2. In the first of them, in $VG_3V$, the bare vertex interaction scatters the system virtually into the $q\bar{q}\,g$-space, where it propagates under impact of the full interaction until a second vertex interaction scatters the system into the $q\bar{q}$-space. The gluon can be absorbed on the same quark line as it was emitted, which contributes to the effective quark mass as illustrated by diagram $U_{1,2}$ of Fig. 2. The gluon absorbed by the other quark as in diagram $U_{1,1}$ of Fig. 2 providing a quark-antiquark interaction which cannot change the quark flavor. It includes all fine and hyperfine interactions, see for instance [14]. The second term in Eq.(11), the annihilation interaction $U_a = VG_3VG_2VG_3V$, potentially provides an interaction between different flavors and is illustrated in diagram $U_{1,0}$

of Fig. 2. As a net result the interaction scatters a quark with helicity $\lambda_q$ and four-momentum $p$ into a state with $\lambda_q'$ and $p'$.

The knowledge of $\psi_b$ is sufficient to retrieve all desired Fock-space components of the total wave-function. The key is the upwards recursion relation Eq.(9). Obviously, one can express the higher Fock-space components $\langle n|\Psi\rangle$ as functionals of $\psi_{q\bar{q}}$ by a finite series of quadratures, *i.e.* of matrix multiplications or of momentum-space integrations. One need not solve an other eigenvalue problem. This is quickly shown by way of example, by calculating the probability amplitude for a $|gg\rangle$- or a $|q\bar{q}\,g\rangle$-state in an eigenstate of the full Hamiltonian. The first two equations of the recursive set in Eq.(9) are

$$\langle 2|\Psi\rangle = G_2\langle 2|H_2|1\rangle\langle 1|\Psi\rangle, \tag{19}$$

$$\text{and} \quad \langle 3|\Psi\rangle = G_3\langle 3|H_3|1\rangle\langle 1|\Psi\rangle + G_3\langle 3|H_3|2\rangle\langle 2|\Psi\rangle. \tag{20}$$

The sector Hamiltonians $H_n$ have to be substituted from Eqs.(12) and (16). In taking block matrix elements of them, the formal expressions are simplified considerably since most of the Hamiltonian blocks in Fig. 1 are zero matrices. One thus gets simply $\langle 2|H_2|1\rangle = \langle 2|VG_3V|1\rangle$ and therefore $\langle 2|\Psi\rangle = G_2VG_3V\langle 1|\Psi\rangle$. Substituting this into Eq.(19) gives $\langle 3|\Psi\rangle = G_3V\langle 1|\Psi\rangle + G_3VG_2VG_3V\langle 1|\Psi\rangle$. These findings can be summarized more succinctly as

$$|\psi_{gg}\rangle = G_{gg}VG_{q\bar{q}\,g}V\,|\psi_{q\bar{q}}\rangle, \tag{21}$$

$$\text{and} \quad |\psi_{q\bar{q}\,g}\rangle = G_{q\bar{q}\,g}V\,|\psi_{q\bar{q}}\rangle + G_{q\bar{q}\,g}VG_{gg}VG_{q\bar{q}\,g}V\,|\psi_{q\bar{q}}\rangle. \tag{22}$$

The finite number of terms is in strong contrast to the infinite number of terms in perturbative series. Iterated resolvents sum the perturbative series to all orders in closed form.

## 3.    DISCUSSION AND PERSPECTIVES

All of the above relations are exact and hold for an arbitrarily large $K$ [10]. They hold thus also in the continuum limit, where the resolvents are replaced by propagators and the eigenvalue problems become integral equations. The effective interaction is very simple in direct consequence of the structure of the gauge field Hamiltonian with its many zero matrices. No particle cut-off is required, and no assumption is made on the size of the coupling constant. The appoach is strictly non-perturbative. Instead of the inverting a huge matrix as in the Tamm-Dancoff approach, one has to invert only the comparatively small sector Hamiltonians $H_n$. The instantaneous interactions can be included easily ex post by the rule that every intrinsic line in graph in Fig. 2 has to be suplemented with the corresponding instantaneous line [10]. Once the general structure of the effective interaction has been formulated, one can proceed with simplifying assumptions. For example, one can replace the full by the free propagators to get a selected set of perturbative diagrams. The major effort of the ongoing work [21] is to find an approximation scheme which combines rigor with simplicity. Unfortunately the limited space prevents giving more details.

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